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Anomaly-Induced Gauge Unification and Brane/Bulk Couplings in Gravity-Localized Theories

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Abstract

It has recently been proposed that gravity-localized compactifications can generate the required gauge hierarchy without the need for hierarchically large extra spacetime dimensions. In this paper, we show that gauge coupling unification arises naturally in such scenarios as a result of the anomaly induced by the rescaling of the wavefunctions of the brane fields. Thus, "anomaly-induced" gauge coupling unification can easily explain the apparent low-energy gauge couplings in gravity-localized compactifications. However, we also point out a number of phenomenological difficulties with such compactifications, including an inability to accommodate the GUT scale and the electroweak scale simultaneously. We also show that brane/bulk couplings in this scenario are generically too small to be phenomenologically relevant. Finally, we speculate on possible resolutions to these puzzles.

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1 Introduction

One of the most surprising theoretical developments of the past few years has been the realization that the fundamental high energy scales of physics are not immutable, and that they can be altered in the presence of extra spacetime dimensions. Specifically, it has been shown that extra spacetime dimensions have the potential to lower the fundamental GUT scale [1], the fundamental Planck scale [2], and the fundamental string scale [3]; indeed, each of these scales can be lowered to potentially accessible energy scales in the multi-TeV range. Despite their similarities, however, these scenarios have important differences. The GUT scale-lowering scenario of Ref. [1] utilizes extra "in-the-brane" dimensions that are roughly of the same order as the (lowered) fundamental scale of the higher-dimensional theory. Thus, no hierarchy is introduced. By contrast, the Planck scale-lowering scenario of Ref. [2] requires extra "off-the-brane" dimensions that are hierarchically larger than the higher-dimensional fundamental scale. Thus, the hierarchy between the weak scale and four-dimensional Planck scale is not explained, but rather reformulated in a geometric context as a new hierarchy between the weak scale and the scale of extra dimensions. Nevertheless, as discussed in Ref. [1], it is possible to combine all three of these scenarios in a consistent way within the framework of Type I string theory. Thus, combining these scenarios, a consistent picture of reduced gauge and gravitational energy scales emerges.

Recently, a new proposal [4] has been made for generating the Planck-scale/weak-scale hierarchy without the use of large extra dimensions, but rather as a result of gravity localization. As such, this scenario explains the apparent weakness of gravity without recourse to large extra dimensions, and provides an interesting alternative to the large-dimension scenario of Ref. [2]. The basic idea is as follows. As shown in Ref. [4], gravity localization emerges naturally in a D-brane context when the spacetime gravitational effects of the D-brane itself are taken into account. Such gravity localization arises because the presence of the D-brane induces the spacetime metric to accrue a scale factor ("warp factor") which is a falling exponential function of the distance along the dimension perpendicular to the brane. Thus, the graviton is essentially "bound" or localized to the D-brane. By imagining that the Standard Model is restricted to a second D-brane whose position is shifted relative to the first, one finds that a hierarchically small scale factor is generated for the metric on the second brane. This in turn requires a rescaling of the fields on the Standard-Model brane, which has the net effect of generating an exponential hierarchy between the mass scales on the Standard-Model brane and the fundamental (higher-dimensional) mass scales. For example, a TeV-sized electroweak scale can be generated on the Standard-Model brane even when this brane is shifted by only a small amount (in Planck-scale units) from the original localization brane. Various generalizations and extensions of this scenario have been considered in Refs. [6, 7, 8, 9, 10]; likewise, earlier solutions involving different "warp factors" can be found in Ref. [11].

While this scenario elegantly generates the desired hierarchy between the Planck scale and the electroweak scale, certain features are left unexplained. One important issue, for example, is to explain how gauge unification might arise in such a context. This issue is particularly pressing for the following reason. In such a gravity-localized scenario, the presence of an exponential warp factor requires that the Standard-Model gauge groups correspond to parallel, coincident D-branes. Indeed, because of the exponential warp factor, even small relative displacements amongst these Standard-Model branes could have potentially large unwanted effects. However, for simplicity, it is natural to expect that the gauge couplings should all take a common, unified value if their corresponding gauge groups arise from parallel, coincident D-branes. Thus, in a gravity-localized compactification, one expects to have only a single, unified gauge coupling for all of the Standard-Model gauge-group factors. Moreover, this unification of gauge couplings should *a priori* arise directly at (or near) the electroweak scale, which is interpreted as the only physical scale on the Standard-Model brane. It then remains to explain why these gauge couplings are experimentally measured to be different.

In this paper, we shall show that gauge coupling unification arises naturally in such scenarios as a result of the anomaly induced by the rescaling of the wavefunctions of the brane fields. Specifically, we shall show that the anomaly produces a contribution to the gauge couplings that splits them in such a way that they appear to have emerged from a traditional high-scale logarithmic unification, or equivalently from a low-scale power-law unification as in Ref. [1].

The success of gauge coupling unification is thus a compelling issue in favor of such gravity-localized scenarios. However, we also point out a number of difficulties with such scenarios, including an inability to accommodate the GUT scale and the electroweak scale simultaneously. Moreover, we also find that brane/bulk couplings in this scenario are generically too small to be phenomenologically relevant. We believe that these issues are generic to the scenario of Ref. [4], and will need to be overcome before a serious investigation of the phenomenology of these scenarios is possible.

Motivated by our results, we then proceed to discuss a possible modification of this scenario which avoids some of these problems. Our modification consists of changing the slope of the warp factor so that the warp factor is maximized rather than minimized on the Standard-Model brane. In this respect our proposal is similar to that of Ref. [5], except that we shall take the radius of the extra dimension to be finite. As we shall see, gauge coupling unification is also easily accommodated in this modified scenario, while brane/bulk couplings take more reasonable sizes and even neutrino masses can be accurately predicted. Proton decay is also sufficiently suppressed, and the scenario as a whole is consistent with an expanding Friedmann-like universe. However, in such a scenario, the generation of the electroweak scale is an important outstanding question. We shall discuss each of these issues in turn, and speculate on some possible resolutions to these puzzles.

2 The framework

We begin by briefly reviewing the scenario of Ref. [4]. This will also enable us to establish our physical and notational conventions, which differ from those of Ref. [4] in some significant ways. It will also enable us to introduce our modified scenario with a flipped warp factor.

2.1 General setup

As in Ref. [4], we take spacetime to be five-dimensional, with the fifth dimension compactified on a $\mathbb{Z}_2$ orbifold of radius R. Thus, the fifth dimension is essentially a line interval parametrized by a coordinate y stretching over the finite interval $0 \leq y \leq \pi R$. If required, one can then formally extend the y-coordinate to all real values using the orbifold symmetry relations

$$y \;\approx\; y + 2\pi R~, \qquad y \;\approx\; -y \;\approx\; 2\pi R - y~. \tag{2.1}$$

We also assume the presence of two D3-branes, one located at each orbifold fixed point, as well as a bulk cosmological constant $\Lambda < 0$. In Ref. [4], the D3-brane located at $y = \pi R$ is presumed to be the one containing the Standard Model. Thus, just as in Ref. [4], the classical action describing this situation is given by

$$\begin{aligned} S \;=\; \int d^4x \Big\{ & \sqrt{-g^{(0)}}\,(\mathcal{L}^{(0)} - V^{(0)}) \;+\; \sqrt{-g^{(\pi R)}}\,(\mathcal{L}^{(\pi R)} - V^{(\pi R)}) \\ & +\; \int_{-\pi R}^{\pi R} dy\, \sqrt{-G}(-\Lambda + 2M^3 R^{(5)}) \Big\} \end{aligned} \tag{2.2}$$

where G is the bulk five-dimensional metric (with negative determinant); $R^{(5)}$ is the five-dimensional curvature derived from G; $g^{(0)}$ and $g^{(\pi R)}$ are the induced four-dimensional metrics on the D3-branes; and M is the fundamental five-dimensional mass scale in the bulk. The goal is then to solve the corresponding Einstein field equations for the five-dimensional bulk metric G.

If, as in Ref. [4], we take a trial solution of the form

$$ds^2 \;=\; e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu \;+\; dy^2~, \tag{2.3}$$

then it is straightforward to derive two differential equations for the unknown function $\sigma(y)$. It turns out [4] that these differential equations do not have solution unless $V^{(0)}$, $V^{(\pi R)}$, and Λ are related according to

$$V^{(0)} = -V^{(\pi R)} = 24M^3 k~, \qquad \Lambda = -24M^3k^2 \tag{2.4}$$

where k is an arbitrary mass scale. In terms of k, the two differential equations then take the form [4]

$$\left(\frac{d\sigma}{dy}\right)^2 = k^2~, \qquad \frac{d^2\sigma}{dy^2} \;=\; -\,\frac{2k^2}{\Lambda}\left[V^{(0)}\delta(y) + V^{(\pi R)}\delta(y-\pi R)\right]~. \tag{2.5}$$

The next step is to solve these differential equations for $\sigma(y)$. In Ref. [4], the solution is taken to be

$$\sigma(y) \;=\; \begin{cases} ky & 0 \leq y \leq \pi R \\ k(2\pi R - y) & \pi R \leq y \leq 2\pi R \;. \end{cases} \tag{2.6}$$

Given this, one can then calculate the effective four-dimensional Planck mass $M_{\rm Planck}$ directly from the five-dimensional curvature term in (2.2) by inserting (2.6) into the metric (2.3) and integrating over the fifth dimension. This yields the result [4]

$$M_{\rm Planck}^2 \;=\; 2M^3 \int_0^{\pi R} dy\, e^{-2ky} \;=\; \frac{M^3}{k}\left(1 - e^{-2\pi k R}\right) \;, \tag{2.7}$$

on the basis of which the authors of Ref. [4] are compelled to choose M and k to be of roughly the same order of magnitude as $M_{\rm Planck}$.

2.2 Orbifold symmetries

In this paper, we wish to consider two distinct scenarios. The first will be the above scenario, with k taken to be a positive quantity, while the second will be the scenario in which k is taken to be negative. However, taking k negative in the above solution leads to certain subtleties which are ultimately spurious, and which disguise the physics we hope to discuss. Therefore, before proceeding further, we shall find it useful to make one additional change in the solution (2.6).

Of course, (2.6) is a perfectly valid solution to the differential equations (2.5). Moreover, as noted in Ref. [4], one is always free to add an arbitrary y-independent constant σ_0 to the solution in (2.6), for this simply amounts to an overall constant rescaling of the four-dimensional metric which in turn amounts to a rescaling of the four-dimensional spacetime coordinates. Such a rescaling therefore has no net physical effect, and in particular does not change the values of physical quantities such as $M_{\rm Planck}$. Indeed, the choice of σ_0 is analogous to a gauge choice, since the physics is ultimately invariant under changes in σ_0, and all σ_0-dependence ultimately cancels in calculations of physical quantites. However, just as with gauge theories, certain choices of σ_0 can make certain features of the calculation more transparent than others.

Although $\sigma_0 = 0$ is the choice taken in Ref. [4], in this paper we shall use a different choice for σ_0. Our choice is motivated by the fact that (2.6) by itself does not make manifest the full orbifold symmetries of the theory. In order to see this, let us consider the following simultaneous transformations

$$\begin{cases} y \to y + \pi R \\ k \to -k \;. \end{cases} \tag{2.8}$$

The first transformation simply amounts to a translation of the y-coordinate by half of the full length of the orbifold. Using the orbifold relations (2.1), we see that $y = 0$

is mapped to $y = \pi R$, while $y = \pi R$ is mapped to $y = 2\pi R$, which is equivalent to $y = 0$. Thus, the mapping $y \to y + \pi R$ simply amounts to exchanging the positions of the two D3-branes. Likewise, from (2.4), it is clear that changing $k \to -k$ also amounts to exchanging the role of the two branes. Thus, we see that the simultaneous transformations given in (2.8) should have no net effect, and should therefore be a symmetry of the theory. Unfortunately, the solution given in (2.6) does not manifest this symmetry. Indeed, rather than remaining invariant under (2.8), we see that

$$\sigma(y + \pi R, -k) \;=\; \sigma(y, k) - \pi k R \;. \tag{2.9}$$

Thus, the solution for $\sigma(y)$ given in (2.6) forces us to perform an overall shift in the absolute value of σ each time we perform the symmetry operations (2.8).

Even though such an overall shift is unphysical, it is awkward to deal with a solution that requires compensating rescalings under orbifold symmetry shifts. This will be particularly troublesome when we choose to consider k to be negative rather than positive. In order to remedy this situation, let us therefore consider a slightly more general solution of the form

$$\sigma(y) \;=\; ky + k\sigma_0 \;, \qquad 0 \leq y \leq \pi R \tag{2.10}$$

where σ_0 is an arbitrary constant to be determined. By the orbifold relations (2.1), this implies $\sigma(y) = k(2\pi R - y) + k\sigma_0$ in the range $\pi R \leq y \leq 2\pi R$. Indeed, since σ_0 is presumed independent of y, this new solution will also satisfy the same differential equations (2.5). Demanding invariance under (2.8), we can then solve for σ_0, yielding the unique result $\sigma_0 = -\frac{1}{2}\pi R$. We therefore conclude that the modified solution

$$\sigma(y) \;=\; \begin{cases} ky - \frac{1}{2}\pi k R & 0 \leq y \leq \pi R \\ k(2\pi R - y) - \frac{1}{2}\pi k R & \pi R \leq y \leq 2\pi R \end{cases} \tag{2.11}$$

not only satisfies the differential equations (2.5), but also exhibits the required invariance under the full orbifold symmetry relations (2.8).

Even though the solution (2.11) is not physically different from that in (2.6), the passage from (2.6) to (2.11) does induce a change in perspective which can often prove useful. As an example of this, let us repeat the calculation of the effective four-dimensional reduced Planck mass M_{Planck} using the shifted solution (2.11) for the five-dimensional metric. Following the same procedure as in (2.7), we now obtain the result

$$M_{\text{Planck}}^2 \;=\; 2M^3 \int_0^{\pi R} dy\, e^{-2ky + \pi k R} \;=\; \frac{2M^3}{k} \sinh\left(\pi k R\right) \;. \tag{2.12}$$

Note that, unlike (2.7), this result for M_{Planck} is now invariant under $k \to -k$. It is desirable to have this invariance under $k \to -k$ because we have integrated over the full fifth dimension when calculating M_{Planck}, and therefore we are not distinguishing

which D3-brane is located in which position. Thus, having $M_{\rm Planck}$ be invariant under $k \to -k$ more closely reflects the inherent symmetries of the system.

Although (2.7) and (2.12) differ by only an overall scale factor, their physical interpretations are different. Whereas previously the authors of Ref. [4] were forced to take M and k to be near $M_{\rm Planck}$, we now see from (2.12) that $M_{\rm Planck}$ can take its correct apparent four-dimensional value even when M and k are taken substantially smaller. This arises because of our choice of taking $\sigma_0 \neq 0$, since M and k are ultimately σ_0-dependent quantities. For example, taking $M = k = 10$ TeV as a typical small reference fundamental mass scale, we find that the apparent value of $M_{\rm Planck}$ can be accommodated simply by taking $kR \approx 21$. Alternatively, if we choose $M \approx 10^{10}$ GeV (as might be preferred on the basis of solving the gauge hierarchy problem as in Ref. [4]), we would require $kR \approx 12$. However, in order for our classical gravity treatment of the brane system to be consistent, we actually must require that $k/M \ll 1$; otherwise the curvature terms in the effective Lagrangian cannot be neglected. Depending on the values of the ratio k/M, the above estimates for kR will change as well. We shall discuss this point in subsequent sections.

Thus, we see that we can generate a sufficiently high Planck scale, even with fundamental bulk mass scales M and k near the electroweak range, simply by taking the fifth dimension to have a radius that is also near the electroweak range. It is easy to interpet this result physically. The apparent four-dimensional Planck mass can be large (even when the fundamental physical scales M and k are small) because the effective *volume* $L_{\rm eff}$ of the fifth dimension can be large (even though the compactification radius R is relatively small). Specifically, we find

$$M_{\rm Planck}^2 ~=~ M^3 \, L_{\rm eff} ~, \tag{2.13}$$

where

$$L_{\rm eff} ~=~ 2 \int_0^{\pi R} dy \, e^{-2ky+\pi kR} ~=~ 2\, k^{-1} \sinh(\pi kR) ~\gg~ R ~. \tag{2.14}$$

Thus, the "warp factor" of the bulk metric has enhanced the small radius into a large volume, thereby enabling the effective four-dimensional Planck mass $M_{\rm Planck}$ to be hierarchically larger than the five-dimensional fundamental scale M.

Of course, we stress again that there is ultimately no physical distinction between the results in (2.7) and (2.12). Rather, the change in metric from (2.6) to (2.11) has merely red-shifted our definitions for the mass scales (M, k) in the bulk from the four-dimensional Planck scale (as in Ref. [4]) to to scales that are much lower. Indeed, M and k are ultimately unphysical quantities, depending on our particular choice for the overall absolute scale factor of the five-dimensional metric. Only $M_{\rm Planck}$, as well as other mass scales on the Standard-Model brane, are truly physical. Thus, despite recent erroneous claims in the literature [20], there is no physical distinction between (2.7) and (2.12) or any other version of this result which involves an overall constant rescaling of the five-dimensional metric. However, as we have seen, the conventions we have established here more naturally reflect the symmetries in the system.

2.3 Two scenarios: $k > 0$ and $k < 0$

We shall be considering two distinct scenarios in this paper. The first, as described above, corresponds to taking k positive. The five-dimensional metric is given in (2.11), and the Planck mass is given in (2.12). We shall discuss the appropriate values of M, k, and kR in subsequent sections, but it is natural to think of M and k as being substantially below the usual Planck scale (perhaps even as low as the TeV-range), and $kR \approx \mathcal{O}(10 - 20)$. We stress, however, that it is generally necessary to have a small hierarchy $k/M \ll 1$ in order to justify our classical treatment wherein we neglected the curvature terms in the effective Lagrangian. This can change the appropriate values of M and kR.

By contrast, the second scenario that we shall discuss corresponds to taking k negative. Note that it is immediately apparent from the above discussion that there is nothing that compels us to consider k to be a positive quantity. Indeed, the above solutions remain completely valid if we consider k to be negative rather than positive. Moreover, as we have seen, the transformation $k \to -k$ is *not* a symmetry of the theory unless simultaneously accompanied by the shift $y \to y + \pi R$ (which we will *not* do). Thus, the solution with $k < 0$ is physically distinct from the solution with $k > 0$. As evident from (2.4), the replacement $k \to -k$ exchanges the signs of the brane potentials, so that the brane at $y = \pi R$ (which we will continue to identify as the brane containing the Standard Model) becomes the one with *positive* potential, *i.e.*, $V^{(\pi R)} > 0$. In other words, in this scenario the "warp factor" $e^{-2\sigma(y)}$ is *maximized* rather than minimized on the Standard-Model brane at $y = \pi R$. In this respect, the negative-k scenario resembles the toy model of Ref. [5], whose purpose was to illustrate the decoupling effects of an infinitely large extra dimension. Unlike Ref. [5], however, in this paper we shall treat the length of the extra dimension as finite, and study the phenomenological properties of the resulting brane configuration. In a cosmological context, we also remark that the negative-k solution, with its positive-energy Standard-Model D3-brane, would also be consistent with a Friedmann-like expanding universe [7].

Note that this change $k \to -k$ merely changes the sign of the brane potentials, but cannot alter the value of quantities such as the fundamental Planck mass. It is for this reason that we have taken the care to establish our conventions such that M_{Planck} is manifestly invariant under $k \to -k$. If we had remained with the original conventions in (2.6), the change $k \to -k$ would have intrinsically involved a spurious overall blue-shifting that we would have had to subsequently disentangle from the effects of having changed the signs of the brane potentials. Indeed, this type of spurious blue-shift has led previous authors [20] to erroneous conclusions, so we cannot emphasize this point strongly enough.

Thus, in the negative-k solution, we shall work with a bulk metric of the form

$$ds^2 \;=\; e^{2\hat{k}y - \pi\hat{k}R}\,\eta_{\mu\nu}dx^\mu dx^\nu \;+\; dy^2\;, \qquad 0 \le y \le \pi R\;, \tag{2.15}$$

where we have defined $\hat{k} \equiv -k > 0$. The corresponding effective four-dimensional Planck mass is given by

$$M_{\text{Planck}}^2 \;=\; \frac{2M^3}{\hat{k}} \sinh\left(\pi \hat{k} R\right) \;, \tag{2.16}$$

and we shall discuss the particular values of M, $\hat{k}$, and $\hat{k}R$ as we proceed. For example, if we take $M = 10$ TeV and demand the existence of a small hierarchy $\hat{k}/M \approx 10^{-4}$, we then find that the apparent value of M_{Planck} can be accommodated with

$$\hat{k}R \;\approx\; 18\;. \tag{2.17}$$

Of course, depending on the chosen values of M and $\hat{k}$, other values for $\hat{k}R$ remain possible. In all cases we shall continue to identify the D3-brane at $y = \pi R$ as the brane containing the Standard Model.

3 Gauge coupling unification

We begin by discussing the issue of gauge coupling unification on the Standard-Model brane, in both the positive- and negative-k scenarios. As discussed in the Introduction, the very nature of the gravity-localized framework with its rapidly changing warp factor requires that the D-branes that comprise the Standard-Model gauge-group factors be parallel and essentially coincident at $y = \pi R$. This implies, in the most straightforward string embeddings, that the gauge couplings corresponding to the different gauge-group factors be essentially equal to each other at the fundamental energy scale M (which we are imagining to be near the electroweak scale). How then can we explain the different observed values of the gauge couplings?

One idea, of course, is to make use of the mechanism advanced in Ref. [1] — namely, to introduce additional "in-the-brane" spacetime dimensions and invoke power-law running for the gauge couplings as a mechanism producing an accelerated unification. This would then require the introduction of another free parameter, namely the radius of the extra "in-the-brane" dimension, and would explain the observed difference in low-energy gauge couplings as a one-loop effect. While in principle this mechanism works even in the gravity-localized context, in this paper we shall consider something different. Essentially, we shall utilize the extra "*off*-the-brane" dimension involved in gravity localization in order to achieve gauge coupling unification directly, *without* the need for renormalization-group running. This would then be an economical explanation of the observed gauge couplings within the gravity-localized framework.

Ordinarily, it might not seem possible for extra dimensions perpendicular to the Standard-Model brane to affect the gauge couplings on the brane in a group-dependent manner. However, in theories involving a warp factor, this is precisely what occurs. Because of the non-trivial warp factor, it is necessary to rescale the

fields on the Standard-Model brane in order to give them a canonical normalization. However, this rescaling is generally anomalous, and induces a shift in the gauge couplings on the Standard-Model brane. Remarkably, we shall find that this shift in the gauge couplings can precisely account for the observed difference in the gauge couplings at the electroweak scale, even if we assume that their "bare" values are universally coupled to the dilaton and hence unified. Thus, in this framework, we *automatically* have a single, unified tree-level gauge coupling on the Standard-Model brane, and it is only an additional anomaly contribution, induced by the warp-factor rescaling, that gives these gauge couplings the different apparent values they are measured to have. We shall therefore refer to this phenomenon as "anomaly-induced" gauge coupling unification.

3.1 The $k < 0$ scenario

Although our "anomaly-induced" mechanism for gauge coupling unification is general and applies to both the positive-k and negative-k solutions, it will prove simpler to first consider the negative-k solution. Accordingly, let us begin with the five-dimensional bulk metric given in (2.15), where we assume that the Standard-Model brane is located at $y = \pi R$. This then induces a four-dimensional metric on the Standard-Model D3-brane given by

$$ds^2 \;=\; e^{\pi \hat{k} R}\, \eta_{\mu\nu} dx^\mu dx^\nu \;, \tag{3.1}$$

whereupon the kinetic-energy terms of the corresponding D3-brane Lagrangian will take the form

$$\mathcal{L} \;=\; \int d^4x \left(e^{\pi \hat{k} R}\, |D_\mu \Phi|^2 \;+\; e^{3\pi \hat{k} R/2}\, \bar{\Psi} i \gamma^\mu D_\mu \Psi \;-\; \frac{1}{4 g_i^2} \mathrm{Tr}\, F_{\mu\nu,i}^2 \right) \;. \tag{3.2}$$

Here Φ, Ψ, and A_μ respectively represent a complex scalar, Dirac fermion, and gauge field. In order to canonically normalize the kinetic-energy terms in (3.2), each of these fields must be Weyl-rescaled by an amount

$$\Phi \to e^{\Lambda}\, \Phi \;, \qquad \Psi \to e^{3\Lambda/2}\, \Psi \;, \qquad A_\mu \to A_\mu \tag{3.3}$$

where $\Lambda = -\pi \hat{k} R/2$. However, while the Lagrangian is classically invariant under such a Weyl-rescaling, it is well-known that this symmetry is anomalous at the quantum level. In other words, the quantum functional-integral measure does not respect this rescaling symmetry, and the resulting Jacobian determinant leads to an extra term in the Lagrangian of the form*

$$\delta \mathcal{L}_{\text{anomaly}} \;=\; \Lambda \sum_i \frac{\beta(g_i)}{2 g_i^3} \mathrm{Tr}\, F_{\mu\nu,i}^2 \;\;+\; \ldots \tag{3.6}$$

* In order to see why the Weyl anomaly is proportional to the beta-functions (which are usually associated with the breaking of invariance under *scale* transformations), let us assume that we start

where the beta-functions $\beta(g_i)$ are defined in terms of the one-loop beta-function coefficients b_i via

$$\beta(g_i) \;=\; \frac{b_i}{16\pi^2}\, g_i^3 \; . \tag{3.7}$$

Note that in writing (3.6) and (3.7), we have chosen a sign convention such that an asymptotically free theory has $b_i < 0$. We shall also assume that the theory on the brane is the Minimal Supersymmetric Standard Model (MSSM) or some other theory that by itself would be consistent with a conventional high-scale logarithmic unification. Thus, performing the Weyl rescaling (3.3), we find that the kinetic-energy terms on the Standard-Model brane will all have the proper canonical normalizations, but the kinetic-energy terms for the gauge fields now take the form

$$\mathcal{L} + \delta\mathcal{L}_{\text{anomaly}} \;=\; -\,\tfrac{1}{4}\sum_i \int d^4x \left(\frac{1}{g_i^2} + \frac{b_i}{16\pi}\,\hat{k}R\right) \,\mathrm{Tr}\, F^2_{\mu\nu,i} \;+ \ldots\, . \tag{3.8}$$

Assuming that the "bare" couplings g_i are all unified at a common value g_U (as might be determined in a string framework through the vacuum expectation value of the dilaton), this enables us to identify the physical couplings as

$$\left.\frac{1}{g_i^2}\right|_{\text{phys}} \;\equiv\; \frac{1}{g_U^2} + \frac{b_i}{16\pi}\,\hat{k}R \; . \tag{3.9}$$

At first glance, this result might not seem to be satisfactory, for we see that the scale anomaly has failed to generate the expected logarithmic term that would be required for a traditional unification of the gauge couplings. Nevertheless, it turns out that this result still leads to a consistent unification. At a mathematical level, the reason for this coincidence is that the result (3.9) is formally identical to the power-law "accelerated" unification that was already previously discussed in Ref. [1]. Indeed, in the language of Ref. [1], we see that the anomaly has precisely reproduced the case with $\delta = 1$ and $\tilde{b}_i = b_i$. The fact that $\tilde{b}_i = b_i$ then guarantees that the unification of gauge couplings in this scenario is exactly as precise as it would have

with an action in curved space which is invariant under the Einstein and Weyl transformations

$$g'_{\mu\nu} = e^{-2\Lambda} g_{\mu\nu}\;, \quad \Phi' = e^{\Lambda}\Phi\;, \quad \Psi' = e^{3\Lambda/2}\Psi\;, \quad A'_\mu = A_\mu\; . \tag{3.4}$$

When restricted to flat space ($g_{\mu\nu} = \eta_{\mu\nu}$), such an action is invariant under the fifteen-parameter four-dimensional conformal group, which contains, in particular, the scale transformations. A simple way to see that Weyl transformations imply scale transformations is to start with (3.4) and find an equivalent transformation in flat space such that $d^4x\sqrt{\det g'} = d^4x'$. This yields $x' = e^{-\Lambda}x$, and when substituted into the Lagrangian this implies the transformations

$$x' = e^{-\Lambda}x\;, \quad \Phi'(x') = e^{\Lambda}\Phi(x)\;, \quad \Psi'(x') = e^{3\Lambda/2}\Psi(x)\;, \quad A'_\mu(x') = e^{\Lambda}A_\mu(x)\; . \tag{3.5}$$

These are precisely the scale transformations. In particular, under $y \to y + \lambda$, we find using (2.11) that $x \to e^{\hat{k}\lambda}x$.

been in the MSSM. Indeed, taking $g_U \approx 0.7$ (which is the usual unified coupling in the MSSM) and $kR \approx 18$, we find that we obtain exactly the same couplings g_i at the fundamental scale $M = 10$ TeV as we would have obtained in the MSSM! Thus, we see that the scale anomaly yields precisely the correct "threshold" corrections that split the observed gauge couplings by an amount that simulates the effects of a conventional logarithmic running over fourteen orders of magnitude. In this respect, our anomaly-induced gauge coupling unification is similar in spirit to the "mirage unification" proposals of Ref. [12].

Although the result (3.9) is formally identical to the power-law unification discussed in Ref. [1], we stress that (3.9) is *not* to be interpreted as resulting from a higher-dimensional running of gauge couplings. Rather, the second term in (3.9) arises from a quantum anomaly, and does not involve the contributions of any Standard-Model Kaluza-Klein states. Nevertheless, it is remarkable that the anomaly has generated precisely the term which can unify the gauge couplings, even without Kaluza-Klein excitations for the Standard-Model gauge or matter fields. This ultimately arises because the anomaly term gives contributions that are proportional to the one-loop beta-function coefficients of the theory living on the Standard-Model D3-brane.

It is also possible to understand this result as a blue-shifting effect on the Standard-Model D3-brane. To see this, let us imagine running the gauge couplings from the fundamental scale M down to their observed values at the Z-scale M_Z. Adding this one-loop running contribution to our result (3.9) then yields

$$\frac{1}{g_i^2(M_Z)} ~=~ \frac{1}{g_U^2} ~-~ \frac{b_i}{8\pi^2}\ln\left(\frac{M_Z}{M}\right) ~+~ \frac{b_i}{16\pi}\,\hat{k}R ~, \tag{3.10}$$

which can be rewritten in the form

$$\frac{1}{g_i^2(M_Z)} ~=~ \frac{1}{g_U^2} ~-~ \frac{b_i}{8\pi^2}\ln\left(\frac{M_Z}{Me^{\pi\hat{k}R/2}}\right) ~. \tag{3.11}$$

Consequently, identifying $M_{\rm GUT} \equiv Me^{\pi\hat{k}R/2}$, we see that the usual GUT scale $M_{\rm GUT} \approx 10^{16}$ GeV can be obtained from the fundamental scale $M \approx 10$ TeV via the *blue-shifting* induced by the warp factor in the metric. Indeed, as stated above, all that is required is a value $\hat{k}R \approx 18$, which is the same value as required in order to generate the correct Planck mass in (2.16). Thus, through this blue-shifting effect, we see that a unified gauge coupling at the low scale $M = 10$ TeV is consistent with the observed values of the low-energy gauge couplings. Of course, in the above equations it is not really necessary to identify the ultraviolet cutoff with the fundamental physical scale M in the bulk. However, without knowing the full underlying theory, it is natural to identify these two scales for simplicity, and we shall continue to do so in what follows.

One interesting consequence of (3.9) is that the radius has a critical value R^* above which the asymptotically free gauge couplings on the Standard-Model brane

become large and ultimately diverge. In the case of the $SU(3)$ coupling, this occurs at the critical value

$$\hat{k}R^* \;=\; \frac{16\pi}{|b_3|\, g_3^2(M)}\;. \tag{3.12}$$

Assuming the MSSM matter content and the unified gauge coupling $g_3(M) = g_U \approx 0.7$, we find the critical radius $\hat{k}R^* \approx 30$. Note that similar phenomena also appear in Type I strings [13] or in M-theory [14]. Of course, it is natural to expect the appearance of a maximum critical radius in any case, since an increase in the radius implies an increase in the corresponding warp factors, which ultimately throws part of the brane/bulk system into a non-perturbative regime.

Given that we have chosen to identify the scale M in (3.11) with the fundamental bulk mass scale, we now see that there are two simultaneous equations that fix the Planck and GUT scales in the negative-k scenario:

$$\begin{cases} M_{\text{Planck}}^2 \;\approx\; (M^3/\hat{k})\, e^{\pi\hat{k}R} \\ M_{\text{GUT}} \;=\; M\, e^{\pi\hat{k}R/2}\;. \end{cases} \tag{3.13}$$

Eliminating R, we obtain

$$M_{\text{Planck}} \;=\; \sqrt{\frac{M}{\hat{k}}}\, M_{\text{GUT}}\;. \tag{3.14}$$

This equation directly relates the fixed physical scales M_{GUT} and M_{Planck} to the ratio $M/\hat{k}$, and holds generically in any such gravity-localized scenario regardless of the overall scale factor for the five-dimensional bulk metric. Thus, in order to successfully reproduce† the values of M_{Planck} as well as M_{GUT} in the negative-k scenario, we see that we must introduce a small hierarchy $M/\hat{k} \approx 10^4$. This justifies our earlier choice in deriving (2.17). Fortunately, this small hierarchy is also compatible with the restriction $\hat{k} \ll M$ that permitted us to trust the classical gravity approximation in Sect. 2 wherein we neglected the curvature R^2 terms in the effective five-dimensional Lagrangian.

Note that the above results can be easily generalized to include the running of other parameters in the Lagrangian of the Standard-Model D3-brane. In order to be specific, let us consider a theory containing gauge fields, fermions Ψ, and complex scalars Φ, with a Lagrangian

$$\mathcal{L} \;=\; \int d^4x\, \left[-\frac{1}{4g^2}\text{Tr}F_{\mu\nu}^2 + \bar{\Psi}(i\gamma^\mu D_\mu - m_\psi)\Psi + |D_\mu\Phi|^2 - m_\phi^2\Phi^\dagger\Phi - \frac{\lambda}{4}(\Phi^\dagger\Phi)^2 + \ldots \right] \tag{3.15}$$

where the ellipses denote other terms irrelevant for the present discussion. Under the Weyl rescaling (3.3), the mass terms explicitly break the classical scale invariance

† In passing, we remark that this also provides another solution to a long-standing problem [15] in the phenomenology of string theory, namely to find a way of reconciling the difference between M_{GUT} and M_{Planck}.

and the path-integral measure is anomalous. This leads to an additional anomaly-generated contribution to the Lagrangian given by

$$\begin{aligned}\delta\mathcal{L}_{\text{anomaly}} &= \Lambda\left[\frac{\beta(g)}{2g^3}\,\mathrm{Tr}\,F_{\mu\nu}^2 + m_\psi(1+\gamma_{m_\psi})\bar{\Psi}\Psi + m_\phi^2(2+\gamma_{m_\phi^2})\Phi^\dagger\Phi - \frac{1}{4}\beta(\lambda)(\Phi^\dagger\Phi)^2\right]\\ &= \Lambda\mu\frac{\partial}{\partial\mu}\mathcal{L}\ ,\end{aligned}\tag{3.16}$$

where γ_{m_ψ} (respectively γ_{m_ϕ}) is the anomalous dimension of Ψ (respectively Φ). Note that in the last line, the derivative acts only on the parameters $g(\mu)$, $m(\mu)$, and $\lambda(\mu)$, which depend on the renormalization scale μ according to

$$\begin{aligned}&\mu\frac{\partial g}{\partial\mu} = \beta(g)\ , \qquad \mu\frac{\partial\lambda}{\partial\mu} = \beta(\lambda)\ ,\\ &\mu\frac{\partial m_\psi}{\partial\mu} = -m_\psi(1+\gamma_{m_\psi})\ , \qquad \mu\frac{\partial m_\phi^2}{\partial\mu} = -m_\phi^2(2+\gamma_{m_{\phi^2}})\ .\end{aligned}\tag{3.17}$$

Thus, for every renormalized parameter $X \equiv (g, m, \lambda, ...)$ in the Standard-Model Lagrangian, we can write the equation

$$\mu\frac{\partial X}{\partial\mu} = \frac{\partial X}{\partial\Lambda}\ .\tag{3.18}$$

This yields the solution $X = X(\mu e^\Lambda)$, where the functional dependence of X is fixed by the usual renormalization-group equations. Since μ always appears in the combination μ/M, this explains why the apparent ultraviolet cutoff for the gauge coupling running is red-shifted from M_{GUT} to $M_{\text{GUT}}e^{-\Lambda}$. This also demonstrates that the same phenomenon actually occurs for *all* running quantities in the Standard-Model Lagrangian.

This last result is highly non-trivial. Of course, it is clear that the rescaling induced by the metric produces corresponding rescaling of the bare (classical) mass parameters in our D3-brane Standard-Model Lagrangian. However, the unification scale M_{GUT} is not a classical mass scale, but rather the result of a one-loop quantum running. Nevertheless, we have shown that even this *quantum* mass scale is rescaled, thanks to the rescaling *anomaly* that automatically accompanies the classical coordinate- (or field-) rescaling. Thus, at both the classical *and* quantum levels, the rescalings induced by the metric have the net effect of successfully rescaling *all* of the mass scales on the brane.

As an example of this, let us consider the running of the top Yukawa coupling y_t in the MSSM. For simplicity, we shall keep only the strong coupling g_3 in the renormalization group equations and take g_3 to be fixed. By integrating the renormalization-group equations, we find the solution

$$\frac{1}{y_t^2(\mu)} = \frac{9}{8g_3^2}\left\{1 - \exp\left(-\frac{g_3^2\hat{k}R}{3\pi}\right)\left[1 - \frac{8g_3^2}{9y_{t_0}^2}\right]\left(\frac{\mu}{M}\right)^{2g_3^2/(3\pi^2)}\right\}\ ,\tag{3.19}$$

where y_{t_0} is the initial value that the Yukawa coupling would have at the scale $Me^{\pi\hat{k}R/2}$. Note that the new exponential factor $\exp(-g_3^2\hat{k}R/(3\pi))$ is small for radii below the critical value (3.12). Therefore, no further constraint on the critical radius arises from considerations of the Yukawa couplings.

We conclude, then, that the gravity-localized scenario with $k < 0$ automatically leads to gauge unification as a result of the anomalous rescaling induced by the gravity-localizing warp factor in the spacetime metric. This unification occurs directly at tree-level (as a result of a "threshold" effect induced by the one-loop anomaly term), and utilizes the same relatively small extra dimensions that already generate the gauge hierarchy. Despite the superficial similarity to the proposal in Ref. [1], we stress that this anomaly-induced unification scenario does not rely on a quantum-mechanical power-law "running" of any sort, and in particular does not involve Kaluza-Klein excitations for any of the Standard-Model gauge or matter fields. Indeed, the Standard Model continues to be restricted to a single set of D-branes (which may therefore be taken to be three-branes). Thus, in this scenario, we see that we can simultaneously explain the weakness of gravity *as well as* the unification of the gauge forces, all as the result of a single extra spacetime dimension whose size is relatively close to the fundamental physical scales in the theory. Moreover, this occurs without large mass scales on the brane or in the bulk. On the other hand, as we shall see, this scenario has difficulty explaining the origin of the electroweak symmetry-breaking scale. We shall discuss this issue in Sect. 4.

3.2 The $k > 0$ scenario

The above mechanism for anomaly-induced gauge coupling unification also applies to the positive-k scenario. We simply algebraically replace $\hat{k}$ in the above expressions with $-k$, and then consider k to be a positive quantity. We thus find the relation

$$M_{\rm GUT} \;=\; Me^{-\pi kR/2} \tag{3.20}$$

which must be satisfied in conjunction with the Planck mass relation given in (2.12). Eliminating R, we find that these equations together imply the relation

$$M_{\rm Planck} \;=\; M_{\rm GUT}\sqrt{\frac{M}{k}}\,e^{\pi kR}~. \tag{3.21}$$

This is the positive-k analogue of the relation (3.14) that we previously found for negative k.

Note that by itself, the relation (3.20) generally requires $M > M_{\rm GUT}$. In other words, although our gauge coupling unification mechanism successfully produces the expected values of gauge couplings at the fundamental mass scale M, this scale M must *exceed* the usual GUT scale. It is easy to understand physically why this is the case. Looking back at (3.9) and replacing $\hat{k} \to -k$ (with k a positive quantity), we see

that the sign of the anomaly-induced contributions is flipped relative to the negative-k case. However, this is satisfactory as an explanation of the gauge couplings at a scale M when $M > M_{\rm GUT}$; indeed, if we imagine running the three gauge couplings *past* their usual unification point, they begin to split again in opposite directions. Of course, the value of M is itself an unphysical quantity, since it can always be rescaled by introducing an additional overall rescaling factor into the five-dimensional bulk metric. As discussed in Sect. 2, such an overall rescaling does not change the physics. There is therefore nothing improper about having $M > M_{\rm GUT}$. To see this more explicitly, let us imagine introducing an additional overall rescaling factor e^{σ_0} into the five-dimensional metric. We would then find that M is replaced by $e^{\sigma_0/2}M$ in the above expressions, and, for suitable choices of σ_0 we can always bring M below $M_{\rm GUT}$. Thus, our mechanism for gauge coupling unification continues to work, even with $M < M_{\rm GUT}$. However, just as in the negative-k solution, there remains a difficulty making this solution consistent with electroweak symmetry breaking. This issue will be discussed further in Sect. 4.

4 Electroweak symmetry breaking: An open question

In the previous section, we discovered that gravity-localized compactifications automatically predict the correct values of the low-energy gauge couplings as a result of a rescaling anomaly. A natural question that remains to be answered is the origin of the electroweak scale. Let us consider the Higgs potential

$$V(H) \;=\; -\,m_0^2|H|^2 + \lambda|H|^4 \tag{4.1}$$

where the minimum of the potential is located at the value $v_0 \equiv m_0/\sqrt{2\lambda}$. In the positive-$k$ scenario, the mass parameter m_0 (which is assumed to be near the Planck scale) is naturally red-shifted towards the TeV scale. Thus, as proposed in Ref. [4], the physical electroweak symmetry breaking scale $v \approx 246$ GeV is identified as $v \equiv e^{-\pi kR/2}v_0$. In this way one obtains a natural electroweak symmetry breaking scale [4] starting from a higher fundamental scale.

However, in the negative-k scenario, we see that if the Higgs mass parameter m_0 is near the TeV-scale, then the corresponding classical blue-shift rescales the Higgs mass parameter to be near the Planck scale. Thus, in order to obtain the correct Higgs mass parameter at the TeV-scale, we would need to begin with an initial mass parameter $m_0 \approx 10^{-4}$ eV! Curiously, however, this is believed to be the size of the four-dimensional cosmological constant. Thus, we are led to the rather unorthodox idea that perhaps the non-zero cosmological constant triggers electroweak symmetry-breaking. Of course, in such gravity-localized scenarios the cosmological-constant problem is intimately connected with the problem of radius stabilization. Discussions of these issues can be found in Ref. [10].

A second possible way of avoiding these undesirable blue-shifting effects might be to eliminate the "bare" Higgs mass parameter altogether, essentially setting $m_0 = 0$.

Of course, we would still require a mechanism for triggering electroweak symmetry breaking, but here one could conceivably use the Coleman-Weinberg mechanism [19] wherein the required Higgs potential is generated via radiative corrections. This leads to the Coleman-Weinberg effective potential

$$V(H) + \delta V_{\text{anomaly}} \;=\; \lambda\,|H|^4 \;+\; C\,|H|^4 \,\ln\Big(|H|^2/\mu^2\Big) - 2C\Lambda|H|^4 \tag{4.2}$$

where μ is a renormalization scale, C is a model-dependent constant, and λ is now an effective coupling whose value depends on the renormalization scale μ. Note that the last term in the potential (4.2) is the anomalous contribution that arises due to the rescaling of the Higgs fields. As we saw in Sect. 3 for the gauge couplings, this anomaly can viewed as effectively rescaling the renormalization scale $\mu \to \mu e^{\Lambda}$. Thus, although the electroweak symmetry continues to be broken by radiative corrections, the coupling λ is now defined at the renormalized scale $\lambda = \lambda(\mu e^{-\pi\hat{k}R/2})$. However, as discussed in Sect. 3, defining the coupling at a rescaled renormalization point is equivalent to shifting the coupling by a finite amount. In particular, the minimum for a simple $\lambda/4!\phi^4$ theory with $C = \lambda^2/(256\pi^2)$ [19] becomes

$$\langle H\rangle \;\approx\; \mu\, e^{-16\pi^2/(3\lambda(\mu e^{\Lambda}))} \;=\; v_0\, e^{\pi\hat{k}R/2} \tag{4.3}$$

where v_0 is the minimum that would have arisen without the anomaly contribution. Thus, we see that even in the Coleman-Weinberg scenario, the electroweak symmetry breaking scale $v \equiv \langle H\rangle$ is again rescaled. Indeed, this is the quantum analogue of the classical rescaling of m_0. Of course, at higher loops, the blue-shifting may also receive contributions from the anomalous dimensions of quantum fields. Furthermore, just as in the usual Coleman-Weinberg scenario, the presence of a heavy top quark mass continues to effectively destabilize the potential. We therefore leave this issue for further study.

Thus, given this rescaling of v, we see that we now have three simultaneous equations that relate the three physical observables M_{Planck}, M_{GUT}, and v to the three parameters (M, k, and kR) that define our gravity-localized compactification. Assuming $kR \gtrsim \mathcal{O}(10)$, so that we may approximate $2\sinh(\pi kR) \approx \exp(\pi kR)$, we find that these three simultaneous equations take the form

$$\begin{aligned}
k>0: &\qquad \begin{cases} M_{\text{Planck}} = M\sqrt{M/k}\,\exp(\pi kR/2) \\ M_{\text{GUT}} = M\exp(-\pi kR/2) \\ v = M\exp(-\pi kR/2) \end{cases} \\
k<0: &\qquad \begin{cases} M_{\text{Planck}} = M\sqrt{M/\hat{k}}\,\exp(\pi\hat{k}R/2) \\ M_{\text{GUT}} = M\exp(\pi\hat{k}R/2) \\ v = M\exp(\pi\hat{k}R/2)\ . \end{cases}
\end{aligned} \tag{4.4}$$

In principle, it is therefore possible to solve simultaneously in each case. For example, solving the Planck and GUT equations simultaneously in each case yields the

relations (3.21) and (3.14). Likewise, in the positive-k solution, if we take $M \approx k$ for simplicity and simultaneously solve the Planck and electroweak constraints, we obtain the solution $kR \approx 12$, $M \approx 10^{10}$ GeV.

Unfortunately, we now see that the additional GUT constraint is incompatible with both of these conclusions. Indeed, from (4.4), we see that regardless of the sign of k, these constraint equations together imply that

$$M_{\rm GUT} \;\approx\; v \; . \tag{4.5}$$

Clearly, this is patently false, failing by approximately 14 orders of magnitude. We stress again that this conclusion follows directly from the wavefunction rescalings and their associated anomalies, which in turn follow directly from the very nature of the gravity-localized compactifications. While this conclusion might be altered if we are willing to accept a large a hierarchy between v_0 and M, this new hierarchy would have to be exactly as large as the hierarchy between $M_{\rm GUT}$ and v that we are seeking to explain. Thus, we conclude that we cannot simultaneously generate the Planck/electroweak hierarchy *and* explain gauge coupling unification in such gravity-localized compactifications. Indeed, this result holds regardless of the sign of k. This, then, seems to be a major difficulty of the gravity-localization framework.

To some extent, this state of affairs is not surprising. In the $k > 0$ scenario, as advocated in Ref. [4], mass scales on the Standard-Model brane are red-shifted down from a high fundamental scale in the bulk. Thanks to the contribution from the rescaling anomaly, this includes not only the classical (bare) mass scales that appear directly in the Lagrangian of the Standard-Model D3-brane, but also those "quantum" scales (such as the GUT scale) which are generated by quantum effects. Therefore, while this scenario provides an elegant solution to the hierarchy problem by generating the electroweak scale from the Planck scale, this scenario is generally incapable of explaining physics that requires the presence of high scales such as the GUT scale. Our arguments concerning gauge coupling unification in this scenario make this last point particularly explicit. By contrast, the $k < 0$ scenario has a different complexion. Here all mass scales on the Standard-Model D3-brane are subjected to a *blue-shifting* effect which, when coupled with the effects of the rescaling anomaly, simultaneously *raises* the classical and quantum mass scales on the Standard-Model D3-brane. As we have seen, this scenario thus has no trouble accommodating gauge coupling uniifcation, which relies upon having a high GUT scale. Indeed, as we shall shortly see, the entire GUT structure of the Standard Model (such as the correct neutrino masses and proton decay) survives intact. However, this scenario has trouble explaining those features that ordinarily rely on the presence of a low electroweak scale (such as the mass scales appearing in Higgs potential).

Thus, these two scenarios are in some sense complementary, with neither providing a full and simultaneous explanation of the disparate energy scales in the Standard Model. In fact, it would appear that this shortcoming is a generic feature of such gravity-localized compactifications. By their very nature, the effect of gravity local-

ization is to introduce a warp rescaling factor (of whatever sign) into the metric on the Standard-Model brane. However, such a warp rescaling factor is universal, and will affect all Standard-Model mass scales simultaneously. Thus, such warp factors do not seem to have the flexibility that would be required in order to simultaneously explain physics that relies on the existence of two disparate mass scales on the same brane.

5 Brane/bulk couplings and neutrino masses

In this section, we investigate the issue of brane/bulk couplings in gravity-localized compactifications. In traditional extra-dimension compactifications with product spacetimes, such couplings between brane fields and bulk fields have proven to play a crucial role in explaining the possible origin of small numbers such as neutrino masses [16, 17]. Generally, these small numbers emerge thanks to a suppression factor arising as a result of the large volume of the extra dimension. It is therefore important to understand how the sizes of these brane/bulk couplings are modified when gravity is localized and no large radii for the extra dimensions are required.

Although the following considerations are quite general and apply to a variety of situations, for concreteness we shall restrict our attention to the case that is most relevant for neutrino masses. In Refs. [16, 17], it was shown that small phenomenologically viable neutrino masses could be produced in extra-dimension scenarios by considering the right-handed neutrino (a Standard-Model singlet field) to reside in the bulk rather than on the brane containing the Standard Model. To this end, we shall consider the particular case of a coupling between two brane fields (a left-handed neutrino ν_L and a Higgs field H) and a single bulk field (a "right-handed" neutrino field $\Psi \equiv (\psi_1, \bar{\psi}_2)^T$ in the Weyl basis). As in Ref. [16], we shall take Ψ to satisfy the orbifold relations $(\psi_1, \psi_2) \to (\psi_1, -\psi_2)$ under $y \to -y$, as a result of which only ψ_1 can couple to the left-handed neutrino ν_L located on the brane at the orbifold fixed point $y = \pi R$. The relevant terms in the action for this system are then given by

$$\begin{aligned} S &= \int d^4x \sqrt{-g} \left\{ g^{\mu\nu} D_\mu H^\dagger D_\nu H \ + \ e^\mu_a \bar{\nu}_L i \bar{\sigma}^a D_\mu \nu_L \ + \ y_\nu (H \nu_L \psi_1|_{y=\pi R} + \text{h.c.}) \right\} \\ &\quad + \int d^4x \int_{-\pi R}^{\pi R} dy \sqrt{-G} \left(M e^M_A \bar{\Psi} i \gamma^A \partial_M \Psi \right) \ . \end{aligned} \tag{5.1}$$

In the first line we have given the kinetic-energy terms for the Higgs field and the left-handed neutrino, where $g \equiv g^{(\pi R)}$ is the metric on the Standard-Model D3-brane located at $y = \pi R$ and where e^μ_a is the vierbein necessary in order to compensate for the non-flat metric. We have also given the Dirac coupling between the left-handed neutrino, the Higgs field, and the right-handed neutrino, where y_ν is the Yukawa coupling. We shall generally assume $\mathcal{O}(10^{-6}) \lesssim y_\nu \lesssim \mathcal{O}(1)$ in order to reflect the expectation that the neutrino Yukawa couplings are within the ranges already set by their corresponding $SU(2)$ lepton counterparts, with $y_\nu^{(e)} \approx \mathcal{O}(10^{-6})$ for the electron

neutrino and $y_\nu^{(\tau)} \approx \mathcal{O}(1)$ for the tau neutrino. Of course, these values are only meant to serve as approximate guides. Finally, in the second line of (5.1) we have given the bulk kinetic-energy term for the right-handed fermion Ψ, where G is the full bulk five-dimensional metric and M is the overall fundamental mass scale in the bulk.

5.1 The $k > 0$ scenario

In order to see the emergence of a "volume" suppression for the Dirac brane/brane/bulk coupling, the next step is to rescale the fields in the system so that they all have canonically normalized kinetic-energy terms. Let us first do this for the case of the scenario proposed in Ref. [4], where the full metric is given in (2.3) with the solution (2.11). Recalling that e_a^μ and e_A^M scale like $\sqrt{g^{\mu\nu}}$ and $\sqrt{G^{MN}}$ respectively, we find that the effective four-dimensional action describing our system takes the form

$$\begin{aligned} S \;&=\; \int d^4x \left\{ e^{-\pi kR} D_\mu H^\dagger D^\mu H \;+\; e^{-3\pi kR/2} \bar\nu_L i \bar\sigma^\mu D_\mu \nu_L \;+\; e^{-2\pi kR} y_\nu (H \nu_L \psi_1^{(0)} + \text{h.c.}) \right. \\ &\qquad\qquad \left. +\; \frac{4M}{3k} \sinh\left(\frac{3\pi kR}{2}\right) \bar\psi_1^{(0)} i \bar\sigma^\mu \partial_\mu \psi_1^{(0)} \right\} . \end{aligned} \tag{5.2}$$

In (5.2), we have kept only the zero-mode $\psi_1^{(0)}$ for simplicity, as this is sufficient for deducing the effect of the volume factor. We have also integrated over the fifth dimension in order to derive the last term, and we shall henceforth approximate $2\sinh(3\pi kR/2) \approx \exp(3\pi kR/2)$. Thus, in order to canonically normalize the kinetic-energy terms, we must do a Weyl-rescaling of the wavefunctions. This causes our brane/bulk coupling term to take the form

$$\int d^4x \; e^{-3\pi kR/2} \sqrt{\frac{3k}{2M}}\, y_\nu (H \nu_L \psi_1^{(0)} + \text{h.c.}) \;, \tag{5.3}$$

whereupon we see that the effective Dirac neutrino mass m_ν is given by

$$m_\nu \;\approx\; \sqrt{\frac{3k}{2M}}\, y_\nu \langle H \rangle \, e^{-3\pi kR/2} \;. \tag{5.4}$$

We thus obtain essentially the expected result, multiplied by an extra "warp" suppression factor $e^{-3\pi kR/2}$. Unfortunately, the effects of this warp factor are quite severe in the scenario of Ref. [4]: taking $kR \approx 12$ (as appropriate for the solution to the gauge hierarchy problem in Ref. [4]), we find $e^{-3\pi kR/2} \approx 10^{-24}$. Thus, with $\langle H \rangle \approx \mathcal{O}(10^2)$ GeV and $y_\nu \approx \mathcal{O}(10^{-6})$, we find

$$m_\nu \;\approx\; 10^{-24}\, y_\nu \langle H \rangle \;\approx\; 10^{-19} \text{ eV} \;, \tag{5.5}$$

which is far too small to be the correct neutrino mass. Even if we take the extreme case $y_\nu \approx \mathcal{O}(1)$, we find $m_\nu \approx 10^{-13}$ eV, which is still too small to match current

experimental expectations. Of course, for a proper treatment one must include the effects of *all* of the Kaluza-Klein modes of the bulk Ψ field, as in Ref. [16], and diagonalize an infinite-dimensional mass matrix. In the case of the gravity-localized scenario, such a Kaluza-Klein decomposition would presumably follow along the lines of Refs. [5, 8]; however, this is beyond the scope of the present paper. Nevertheless, just at the level of the Dirac mass coupling, it is already apparent that in the scenario of Ref. [4], the warp factor from the metric generically tends to *over-suppress* the brane/bulk couplings. This is therefore an important phenomenological problem for this scenario.

5.2 The $k < 0$ scenario

Let us now see how this result is altered in our modified scenario with $k < 0$. We begin again with the action (5.1), and now substitute the bulk metric given in (2.15). Following the same steps as before, we find that this leads to the effective four-dimensional action obtained by replacing $k \to -\hat{k}$ in (5.2). Because this flips the warp factors in the first line of (5.2) while leaving the second line invariant, the required Weyl-rescaling of the brane-field wavefunctions is flipped while the Weyl-rescaling of the bulk-field wavefunction is unaltered. We thus find that our effective Dirac neutrino mass m_ν is now given by

$$m_\nu \;\approx\; \sqrt{\frac{3\hat{k}}{2M}}\; y_\nu \langle H\rangle \tag{5.6}$$

where M_{Planck} is given by (2.16). Of course, we have already seen in (3.14) that the ratio $\hat{k}/M \approx 10^{-4}$ is fixed by the ratio of the GUT scale to the Planck scale. Taking $y_\nu \approx \mathcal{O}(10^{-6})$ as a rough estimate for the electron-neutrino Yukawa coupling, we thus find

$$m_\nu \;\approx\; 10^{-2}\, y_\nu \langle H\rangle \;\approx\; 10^3 \text{ eV} . \tag{5.7}$$

This too fails to be within the range of the current experimental expectations. Thus, we see that while the scenario of Ref. [4] yields Dirac neutrino masses that are vanishingly small, in our scenario the Dirac neutrino masses are slightly larger than expected.

Of course, as in any model of neutrino masses, the ultimate values of the neutrino masses depend crucially on the chosen values of the corresponding Yukawa couplings. For example, in order to obtain a value $m_\nu \approx 10^{-3}$ eV, we would need a Dirac Yukawa coupling $y_\nu \approx 10^{-12}$. Alternatively, one can relax the condition that the ultraviolet cutoff scale on the Standard-Model D3-brane be the same as the physical scale M in the bulk. Indeed, as discussed in Ref. [6], one can imagine attempting to realize such gravity-localized scenarios within the framework of Type I string theory. In such cases, it is reasonable to assume that our brane ultraviolet cutoff is of the order of the string scale M_I, which in turn can be substantially different from the five-dimensional bulk Planck mass M. For example, taking $M = 10$ TeV and an

ultraviolet cutoff scale $M_I = 10^{10}$ GeV, we can obtain a neutrino mass of 10^{-3} eV for a Yukawa coupling $y_\nu \approx 10^{-6}$. The usual Planck and GUT scales continue to be related provided $\sqrt{M/\hat{k}} \approx 10^8$ and $\hat{k}R \approx 10$, which gives rise to the solutions $\hat{k} \approx 10^{-3}$ eV and $R^{-1} \approx 10^{-4}$ eV. Remarkably, we thus obtain a millimeter-sized extra dimension as in Ref. [2]! Obviously, there are now large hierarchies between the different mass scales in the theory, but this gives an example of how the neutrino problem might be solved. Thus, while a problem exists, it seems that our scenario with $k < 0$ can perhaps be more easily accommodated within a flavour-violating theory of neutrino masses.

We have seen that obtaining Dirac neutrino masses within the required experimental range does not work well for both scenarios. However, since the blue-shifting allows us to bring high-scale mechanisms down to lower energies (such as occurred for gauge coupling unification), it follows that the *usual* seesaw mechanism can also be brought down to low energies in a similar way.

To see this, let us consider the usual seesaw mechanism on the Standard-Model brane. Unlike the previous discussion, we shall here introduce a right-handed neutrino *only* on the brane. The wavefunction of the right-handed neutrino scales just like that for the left-handed neutrino, and after rescaling, the Lagrangian takes the form

$$\int d^4x \;\; y_\nu H \nu_L \psi_1 + M e^{\pi \hat{k} R/2} \psi_1 \psi_1 + \text{h.c.} \;. \tag{5.8}$$

Here ψ_1 is now a purely four-dimensional Weyl spinor on the Standard-Model brane, and M is a Majorana mass for the right-handed neutrino field. Note that only the bare Majorana mass term is rescaled since this term classically breaks the scale invariance. Integrating out the right handed neutrino field then gives rise to a dimension-five operator

$$\frac{y_\nu^2}{M e^{\pi \hat{k} R/2}} \, \nu_L \nu_L H H \tag{5.9}$$

which in turn leads to a neutrino Majorana mass

$$m_\nu \;\approx\; \frac{y_\nu^2 \langle H \rangle^2}{M e^{\pi \hat{k} R/2}} \;\approx\; 10^{-3} \text{ eV} \;, \tag{5.10}$$

where we have used $M = 10$ TeV, $\hat{k}R \approx 18$, and $y_\nu \approx 1$. Thus, for the negative-k scenario, we see that we are able to obtain the required neutrino masses using the usual seesaw mechanism without ever having to introduce a heavy mass scale. This arises because the negative-k scenario, due to its blue-shifting factor, essentially reproduces the usual GUT structure and mass relations on the Standard-Model brane even though the bare mass scales on the Standard-Model brane are small. Indeed, similar arguments can also be used to demonstrate that proton stability is also not a problem in the negative-k scenario.

Note that in the positive-k scenario, the simple seesaw mechanism does not work since the large intermediate mass scale will be red-shifted rather than blue-shifted.

This leads to unaccepatble levels of lepton-number violation at low energies. A similar problem will also arise for proton decay. Thus, we see that the negative-k scenario can more easily accommodate neutrino masses within the required experimental range.

6 Conclusions

In this paper, we have considered gravity-localized compactification scenarios in which a "warp factor" generates the hierarchy between the weak scale and the usual four-dimensional Planck scale. Like the scenario originally proposed in Ref. [4], only one extra dimension is required, and there are no large hierarchies between the scale of extra dimensions and the fundamental physical scale of the theory. Unlike the scenario of Ref. [4], however, in our formulation this scenario involves no high physical scales (either on the brane or in the bulk).

Our main result is that gauge coupling unification emerges naturally in such scenarios, and arises thanks to a rescaling anomaly. This "anomaly-induced" gauge coupling unification thus explains the different values of the low-energy gauge couplings on the Standard-Model brane, and represents a new mechanism for achieving gauge "unification" at reduced energy scales. Because of its generality, requiring only the presence of a non-trivial warp factor, this anomaly-induced unification mechanism may also be applicable to many other similar gravity-localized scenarios that have recently been proposed [9].

Given this result, we then proceeded to investigate the compatibility of the GUT scale and the electroweak symmetry breaking scale. Unfortunately, the results are rather discouraging, generically requiring $M_{\mathrm{GUT}} \approx v$. This is a signal that gravity-localized compactification scenarios generically cannot accommodate widely separated mass scales on a single Standard-Model brane. (By contrast, the Planck scale emerges directly from the bulk where gravity is free to propagate, even if in a restricted manner.) We also pointed out various speculative ideas concerning how this issue might ultimately be resolved, and also considered the sizes of generic brane/bulk couplings in such gravity-localized scenarios.

Overall, despite the difficulties in accommodating the GUT scale and the electroweak scale simultaneously, we feel that the phenomenological prospects of gravity-localization are rich and have hardly been explored. The multitude of possibilities inherent in this framework therefore suggests that this question is worthy of further study.

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